Solution of Cooperative Game based on Particle Swarm Optimization (PSO) with Uncertain Alliance

Zhuangkuo Li [a], Shuipeng Chen [b]

School of Business, Guilin University of Electronic Technology, Guilin 541004, China

[a]237578130@qq.com, [b]863716535@qq.com

Keywords: Cooperative game; uncertainty; Particle swarm optimization; negotiation; alliance income.

Abstract: In traditional solution of cooperative game, it is usually assumed that alliance income is determined or the players agree on the value of alliance income. The income of alliance is often uncertain and the opinions of the players on the value of alliance income are inconsistent. In this case, this paper first describes the benefits of the players when alliance income is uncertain, and establishes the extended model of the cooperative game. Then, considering the rational negotiation and decision game of players, the model is solved in two stages based on the particle swarm optimization algorithm. This paper provides new ideas and methods for solving the cooperative game in uncertainty alliance income.

1. Introduction

Cooperative game is one of the important theoretical methods to solve the payoff allocation in cooperation. It has been more than 70 years since the classical cooperative game theory has been founded by von Neumannn and Morgenstern [1]. And some scholars have proposed many classic cooperative game solutions. Shapley proposed the Shapley value and established axiomatic proof that the cooperative game began to be recognized by the public [2]. Gillies considers the individual rationality of the player and the validity of the alliance, the concept of nuclear is proposed from the perspective of dominance [3]. Aumann et al. regard the formation process of the distribution result as the negotiating process of the game, and propose a negotiation solution, which reflects the rationality of the distribution strategy [4]. The related solution concept also includes the kernel [5] and the nucleolus [6]. Tijs, S. proposed a new solution concept τ-value, and then gave an axiomatization method of τ-value [7,8]. The concept of these classic cooperative game solutions is based on the assumption that the value of the coalition income is determined.

In reality, as Harsanyi, J. C. pointed out, “the player may lack full information about the other players’ payoffs (or even their own), etc. [9]” The alliance building by the players is usually geared to the future and the future payoff of the alliance are often uncertain. Some scholars assume that the players can form a common knowledge of the probability distribution of alliance payoff, and then improve the traditional solution to solve cooperative game problems. For example, Yang, X et al. defined the expected core and the α-optimistic core, and gave an enough and necessary condition that provides a way to find the non-emptiness of the uncertain core [10]. Jinwu Gao et al. was inspired by the Shapley value to propose the concept of uncertain Shapley value as a solution to the uncertain alliance game, and proves the uniqueness of the uncertain Shapley value [11]. Algaba, E et al. proposed Coloured Egalitarian Solution and Coloured Cost Proportional Solution when they studied the profit distribution of different types of transport companies in intermodal transport [12].

Although the above researches have promoted the development of solving cooperative games when the payoff of the alliance is uncertain, there are still many deficiencies. These scholars did not consider the difference in the value of the payoff of the alliance, nor did they describe and analyze the negotiation process of the players. The players have differences in individual experience, individual information, individual rationality, situation judgment, interest appeal, etc. It is difficult to form a consensus on the value of alliance income, and the players have their own judgments. The achievement of the alliance allocation scheme is usually the result of multiple rounds of negotiation, mutual influence, mutual compromise, and final convergence of the bureaucrats based on individual rationality and judgment. At present, there is no in-depth study on how to solve cooperative game when the value of alliance income is uncertain and players' opinions on its value are inconsistent. Based on particle swarm optimization (PSO), this paper analyzes and describes individual adjustment and group convergence behaviors during the formation of distribution schemes from the perspectives of individual decision making and group interaction, and conducts modeling and solving of cooperative games when opinions on the value of alliance income are not consistent.

2. Particle Swarm Optimization

Particle swarm optimization was proposed by Eberhart, R. and Kennedy, J. as a simple model of social learning whose nascent behavior gained popularity as a technique for solving complex optimization problems in a reliable and simple manner [13]. The basic idea of the particle swarm optimization algorithm is to randomly initialize a group of particles in the feasible solution space, and treat each particle as a feasible solution to the problem. The quality of the particle is determined by a preset fitness function. Particles follow the current optimal particle movement in the feasible solution space, and the optimal solution is obtained through generation-by-generation search. In each generation, the particle will track two extremes: so far, the best solutions found by the particle itself and the whole population [14].

It is assumed that in the m-dimensional space, the particle group is composed of n particles, and the position vector x_i of any one of the particles i at time k is represented as $x_{ij}^k = (x_{i1}^k, x_{i2}^k, \cdots, x_{im}^k)^T$, and the velocity vector v_i is represented as $v_{ij}^k = (v_{i1}^k, v_{i2}^k, \cdots, v_{im}^k)^T$. The particle position and the way of velocity update during the evolution are as follows:

$$x_{ij}^{k+1} = x_{ij}^k + v_{ij}^{k+1} \tag{1}$$

$$v_{ij}^{k+1} = w^k v_{ij}^k + c_1 r_1 (pbest_{ij}^k - x_{ij}^k) + c_2 r_2 (gbest_j^k - x_{ij}^k) \tag{2}$$

Eqs. (1) and (2) constitute the basic particle swarm optimization, where $i = 1,2,\cdots,n$, $j = 1,2,\cdots,m$. n is the number of particle swarms, and m is the dimension of the target space. w^k is the inertia weight. c_1 and c_2 are learning factors. r_1 and r_2 are random numbers evenly distributed between (0,1). $pbest_{ij}^k$ is the individual optimal position of the i-th particle in the t-th iteration. $gbest_j^k$ is the overall optimal position in the k-th iteration.

Shi, Y. et al. studied the influence of inertia weight on the performance of the algorithm through experiments [15]. It can be known that the inertia weight w^k determines the degree of inheritance of the previous round of adjustment values, and the analysis indicates that the larger inertia weight is conducive to the global optimization, while the smaller one is favour of local optimization. In order to balance the global search ability and local search ability of particle swarm and improve the speed of search and solution. A nonlinear dynamic improved inertia weighting strategy is adopted, expression of the inertia weight w^k is calculated by

$$w^k = w_e + (w_s - w_e)\exp(-r \times (\frac{k}{k_{\max}})^2) \tag{3}$$

Where k is the current number of iterations, $k_{\max}$ is the maximum number of iterations, w_s , w_e are the initial inertia weight and the termination inertia weight respectively. *r* is the control factor that controls the smoothness of the curve of change of *w* and *k*. It has been verified by predecessors that the average best adaptive value of *r* in the interval (3.0, 4.0) is stable, so *r* = 3.5.

3. Particle Swarm Optimization for Cooperative Game

The process of cooperative game solving is set as the players propose their respective distribution schemes, and then negotiate to constantly adjust the distribution schemes, and finally achieve the convergence of schemes. Each player is abstracted as a particle, each player's allocation scheme is abstracted as the particle's position in space, the adjustment value of the player's allocation scheme is abstracted as the particle's velocity, and the set of all players is abstracted as a group.

Definition 1. In the k round of negotiation, the pre-allocation scheme proposed by player i is set as $x_{i\bullet}^k=[x_{i1}^k,x_{i2}^k\cdots x_{ij}^k\cdots x_{in}^k]$, the pre-allocation vector proposed by all players to player j is set as $x_{\bullet j}^k=[x_{1j}^k,x_{2j}^k,\cdots,x_{nj}^k]^T$, and the pre-allocation scheme proposed by all players constitutes the pre-allocation matrix X^k, that is:

$$X^k=\begin{bmatrix} x_{1\bullet}^k \\ x_{2\bullet}^k \\ \vdots \\ x_{i\bullet}^k \\ \vdots \\ x_{n\bullet}^k \end{bmatrix}=\left[x_{\bullet 1}^k,x_{\bullet 2}^k,\ldots x_{\bullet j}^k \ldots x_{\bullet n}^k\right]=\begin{bmatrix} x_{11}^k & x_{12}^k & \ldots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \ldots & x_{2n}^k \\ \vdots & \ldots & x_{ij}^k & \vdots \\ x_{n1}^k & x_{n2}^k & \ldots & x_{nn}^k \end{bmatrix}$$

Definition 2. Let X_n^k be an n-dimensional decision space, representing a set of decision variables. The individual optimal allocation scheme for player i is $pbest_{i\bullet}^k$, and $pbest_{ii}^k$ is called the individual optimal allocation value, where $pbest_{ii}^k \ge x_{si}^k,\ s=1,2,\cdots,i-1,i+1,\cdots n$ is the mapping $f: X_n \to pbest_{i\bullet}^k$, satisfying:

$$f_i(X,k)=pbest_{i\bullet}^k \tag{4}$$

Firstly, the values which are distributed to the player i by all players are sorted, and $\max x_{\bullet i}^k$, the maximum allocation value, is selected. If the maximum allocation value $\max x_{\bullet i}^k$ is not the allcation value of the player i to itself , the maximum allocation value $\max x_{\bullet i}^k$ is set as the individual optimal allocation value $pbest_{ii}^k$ .Otherwise , the second maximum allocation value is chosen as the individual optimal allocation value $pbest_{ii}^k$ .The allocation vector including $pbest_{ii}^k$ (the allocation scheme it represents gives the best payoff to player i) is set as $pbest_{i\bullet}^k$. Mapping f is a strategy function. In the cooperative game, $pbest_{i\bullet}^k$, the individual optimal pre-allocation scheme, refers to the satisfactory pre-allocation scheme determined by players after comparing all the pre-allocation schemes in the *k*-th game.

Definition 3. The learning factors c_1^k and c_2^k determine the influence of the player's own experience and group experience on the adjustment of the distribution scheme of the player. Larger or smaller learning factors are not conducive to calculation and optimization. A learning factor that changes linearly with the number of iterations is used to control the distribution scheme of players. The early stage focuses on self-learning ability and the later stage focuses on social learning ability [16]. The formula for calculating the linear function of the learning factor is as follows:

$$c_1^k=c_{1s}-(c_{1s}-c_{1e})\times k/m \tag{5}$$

$$c_2^k=c_{2s}-(c_{2s}-c_{2e})\times k/m \tag{6}$$

Where m is the maximum number of iterations, c_{1s} is the initial value of c_1, $c_{1s}=1$, c_{1e} is the final value of c_1, $c_{1e}=0$, c_{2s} is the initial value of c_2, $c_{2s}=0$, c_{2e} is the final value of c_2, $c_{2e}=1$.

Definition 4. The scheme is composed by the mean values of the values which is distributed to the player j by all the players is set as the global optimal allocation scheme $gbest^k$, and the calculation formula of the global optimal allocation value $gbest^k$ is:

$$gbest_{ij}^k=\frac{1}{n}\sum_{i=1}^{n}x_{ij}^k \quad , \ j=1,2,\cdots,n \tag{7}$$

Definition 5. In the process of cooperative game solving, under the restriction of limited rationality, players adjust their allocation schemes according to the information of individual optimal allocation scheme and current global optimal allocation scheme. Therefore, individual rational factor r_1 and global rational factor r_2 are added to the iteration formula, where r_1 and r_2 are random numbers between (0, 1).The gap between the individual optimal allocation scheme $pbest_{i\bullet}^k$ of the player i and the distribution scheme $x_{i\bullet}^k$ of the player i is temporarily referred to as the relative individual optimal gap. The gap between the global optimal allocation scheme $gbest^k$ and the distribution scheme $x_{i\bullet}^k$ of the in-office person i is referred to as a relative global optimal gap. Therefore, the product $r_1\times(pbest_{ij}^k-x_{ij}^k)$ of the individual rational factor and the relative individual optimal gap of the player i can represent the individual experience learning in each iteration. The product $r_2\times(gbest_{ij}^k-x_{ij}^k)$ of the overall rational factor and the relative overall optimal gap can represent the group experience learning in each round of iteration.

Definition 6. The fitness of the cooperative game is set as $g(X,k)$, the process of convergence of the cooperative game pre-allocation scheme is described as $\lim\limits_{k\to k_{end}} g(X,k)=0$, and the fitness function expression is:

$$g(X,k)=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}(gbest_{ij}^k-x_{ij}^k)^2}+\sqrt{\sum_{j=1}^{n}(\max(x_{\bullet j}^k)-\min(x_{\bullet j}^k))^2} \tag{8}$$

4. Construction Model of Cooperative Game

Considering that many things in reality have both certain and uncertain attributes, this paper divides the $v(\mathrm{N})$ of alliance revenue into two parts: deterministic income A (deterministic value) and uncertain income B (uncertain value), that is, $A=0$ .The individual payoff of the player is expressed as $y_i=\alpha_i+\beta_i B$,α_i is the deterministic income distribution of the player i, β_i is the distribution coefficient of the player i for the uncertain income.

The iterative process of particle swarm optimization algorithm is equivalent to the negotiating process of the players in the actual situation. Each round of iterative adjustment of the allocation scheme needs to meet the group rationality, that is, the sum of the benefits allocated to each player in the allocation scheme is equal to the income of the major alliance. In addition to meeting the validity, it is also necessary to satisfy the individual's rationality, that is, the income distribution of the players in the alliance is not less than the income when they do not participate in the alliance.

For player i, the individual rationality is:

$$\alpha_i+\beta_i B\ge v(i) \tag{9}$$

Group rationality is:

$$\sum_{i=1}^{n} y_i = v(N) \tag{10}$$

$$\sum_{i=1}^{n} \alpha_i = A \tag{11}$$

$$\sum_{i=1}^{n} \beta_i = 1, 0 \le \beta_i \le 1 \tag{12}$$

5. Two-stage Solution based on Particle Swarm Optimization

5.1 Solution of Deterministic Income

The deterministic income distribution a_i of the player i is substituted into the particle swarm optimization for cooperative game to solve the problem. For the distribution of deterministic income, within the constraints of satisfying the individual rationality and group rationality, in the k round of negotiations, the pre-allocation scheme proposed by player i is $\alpha_{i\bullet}^k = [\alpha_{i1}^k, \alpha_{i2}^k \cdots \alpha_{ij}^k \cdots \alpha_{in}^k]$, the pre-allocation vector proposed by all players for player j is $\alpha_{\bullet j}^k = [\alpha_{1j}^k, \alpha_{2j}^k, \cdots, \alpha_{nj}^k]^T$, and the pre-allocation scheme proposed by all players constitutes the pre-allocation matrix A^k, i.e.

$$\mathrm{A}^k = \begin{bmatrix} \alpha_{1\bullet}^k \\ \alpha_{2\bullet}^k \\ \vdots \\ \alpha_{i\bullet}^k \\ \vdots \\ \alpha_{n\bullet}^k \end{bmatrix} = \left[\alpha_{\bullet 1}^k, \alpha_{\bullet 2}^k, \ldots \alpha_{\bullet j}^k \ldots \alpha_{\bullet n}^k\right] = \begin{bmatrix} \alpha_{11}^k & \alpha_{12}^k & \ldots & \alpha_{1n}^k \\ \alpha_{21}^k & \alpha_{22}^k & \ldots & \alpha_{2n}^k \\ \vdots & \ldots & \alpha_{ij}^k & \vdots \\ \alpha_{n1}^k & \alpha_{n2}^k & \ldots & \alpha_{nn}^k \end{bmatrix}$$

The row vector in A^k represents the allocation scheme proposed by the players. The initial value $a_{i\bullet}^1$ represents individual characteristics, reflecting individual rational situation, information structure, value orientation and other individual characteristics. The mean value, extreme value, feature vector of A^1 represent group characteristics, reflecting group preference, group information structure, social value orientation and other group characteristics. There is usually a large difference between the row vectors in the initial allocation matrix (representing different appeals for the allocation scheme when the players are initially negotiating). In the round, player i update the distribution scheme to form a new distribution matrix. After a finite round of iterations, if the row vectors are equal, a consistent allocation scheme is formed, and the cooperative game obtains a single-valued solution. If they are not equal, it means that the players still have differences and fail to reach a cooperation.

The specific steps for calculating the deterministic income a_{ij} are as follows:

Step 1 All players choose the game strategy, propose the initial scheme A^1 according to the constraints. Setting the particle inertia weights w_s, w_e, the adjusted coefficients $c_{1s}, c_{1e}, c_{2s}, c_{2e}$, the maximum number of iterations m and other basic initial parameters.

Step 2 The particles calculate the inertia weight w^k according to the equation (3), and calculate the learning factors c_1^k and c_2^k according to the equations (5) and (6), respectively.

Step 3 According to the game rules of the players and the position vector, $pbest_{ij}^k$ is calculated by equation (4).

Step 4 The global optimal allocation scheme $gbest_{ij}^k$ is calculated according to equation (7).

Step 5 Calculate the next position coordinates and fitness. The rational factors r_1 and r_2 are generated randomly between (0,1). w^k, $pbest_{ij}^k$, $mean_{ij}^k$, etc. are substituted into equation (2) to calculate the iterative velocity v_{ij}^{k+1} of the $k+1$ round by combining the current velocity v_{ij}^k. The velocity v_{ij}^{k+1} is substituted into equation (1) to calculate the position coordinate a_{ij}^{k+1} of the particle.

Step 6 Determine the iteration termination condition. The fitness $g(\mathrm{A},k)$ is calculated according to equation (8), and judge whether the fitness is 0. If $g(\mathrm{A},k)=0$, the iteration stops and the optimal position vector is output. If $g(\mathrm{A},k)>0$, judge whether the maximum iteration number is reached; if not, turn to step 2. If the maximum number of iterations is reached, the iterative calculation is stopped, and the final pre-allocation scheme matrix and difference degree are output.

After p rounds of iteration, each row of the matrix was equal, that is, all players reached an agreement on the pre-allocation scheme, and the final result of a_{ij} was obtained:

$$\mathrm{A}^p = \begin{bmatrix} \alpha_{11}^p & \alpha_{12}^p & \cdots & \alpha_{1n}^p \\ \vdots & \ddots & & \vdots \\ \alpha_{n1}^p & \alpha_{n2}^p & \cdots & \alpha_{nn}^p \end{bmatrix}$$

Where $\begin{bmatrix} \alpha_{11}^p & \alpha_{12}^p & \cdots & \alpha_{1n}^p \end{bmatrix} = \cdots = \begin{bmatrix} \alpha_{n1}^p & \alpha_{n2}^p & \cdots & \alpha_{nn}^p \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{bmatrix}$.

5.2 Solution of Uncertain Income

The uncertainty distribution coefficient β_i of the player i is substituted into the particle swarm optimization for cooperative game to solve the problem. For the distribution of uncertain income, within the constraints of satisfying the individual rationality and group rationality, in the k round of negotiations, the pre-allocation scheme proposed by player i is $\beta_{i\bullet}^k = [\beta_{i1}^k, \beta_{i2}^k \cdots \beta_{ij}^k \cdots \beta_{in}^k]$, the pre-allocation vector proposed by all players for player j is $\beta_{\bullet j}^k = [\beta_{1j}^k, \beta_{2j}^k, \cdots, \beta_{nj}^k]^T$, and the pre-allocation scheme proposed by all players constitutes the pre-allocation matrix B^k, i.e.

$$\mathrm{B}^k = \begin{bmatrix} \beta_{1\bullet}^k \\ \beta_{2\bullet}^k \\ \vdots \\ \beta_{i\bullet}^k \\ \vdots \\ \beta_{n\bullet}^k \end{bmatrix} = \begin{bmatrix} \beta_{\bullet 1}^k, \beta_{\bullet 2}^k, \ldots \beta_{\bullet j}^k \ldots \beta_{\bullet n}^k \end{bmatrix} = \begin{bmatrix} \beta_{11}^k & \beta_{12}^k & \ldots & \beta_{1\mathrm{n}}^k \\ \beta_{21}^k & \beta_{22}^k & \ldots & \beta_{2\mathrm{n}}^k \\ \vdots & \ldots & \beta_{\mathrm{ij}}^k & \vdots \\ \beta_{\mathrm{n}1}^k & \beta_{\mathrm{n}2}^k & \ldots & \beta_{\mathrm{nn}}^k \end{bmatrix}$$

The row vector in B^k represents the allocation scheme proposed by the player. The initial value $\beta_{i\bullet}^1$ represents individual characteristics, reflecting individual rational situation, information structure, value orientation and other individual characteristics. The mean value, extreme value, feature vector of B^1 represent group characteristics, reflecting group preference, group information structure, social value orientation and other group characteristics. There is usually a large difference between the row vectors in the initial allocation matrix (representing different appeals for the allocation scheme when the players are initially negotiating). In the round, player i update the distribution scheme to form a new distribution matrix. After a finite round of iterations, if the row vectors are equal, a consistent allocation scheme is formed, and the cooperative game obtains a single-valued solution. If they are not equal, it means that the players still have differences and fail to reach a cooperation.

The specific steps for calculating the uncertainty distribution coefficient β_{ij} are as follows:

Step 1 All players choose the game strategy, substitute the calculated deterministic income a_i into the constraint conditions, propose the initial scheme B^1 according to the new constraint conditions. Resetting the particle inertia weight w_s, w_e, the adjusted coefficients $c_{1s}, c_{1e}, c_{2s}, c_{2e}$, the maximum number of iterations m and other basic initial parameters.

Step 2 The particles calculate the inertia weight w^k according to the equation (3), and calculate the learning factors c_1^k and c_2^k according to the equations (5) and (6), respectively.

Step 3 According to the game rules of the players and the position vector, $pbest_{ij}^k$ is calculated by equation (4).

Step 4 The global optimal allocation scheme $gbest_{ij}^k$ is calculated according to equation (7).

Step 5 Calculate the next position coordinates and fitness. The rational factors r_1 and r_2 are generated randomly between (0,1). w^k, $pbest_{ij}^k$, $mean_{ij}^k$, etc. are substituted into equation (2) to calculate the iterative velocity v_{ij}^{k+1} of the $k+1$ round by combining the current velocity v_{ij}^k. The velocity v_{ij}^{k+1} is substituted into equation (1) to calculate the position coordinate β_{ij}^{k+1} of the particle.

Step 6 Determine the iteration termination condition. The fitness $g(\mathrm{B},k)$ is calculated according to equation (8), and judge whether the fitness is 0. If $g(\mathrm{B},k)=0$, the iteration stops and the optimal position vector is output. If $g(\mathrm{B},k)>0$, judge whether the maximum iteration number is reached; if not, turn to step 2. If the maximum number of iterations is reached, the iterative calculation is stopped, and the final pre-allocation scheme matrix and difference degree are output.

After q rounds of iteration, each row of the matrix was equal, that is, all players reached an agreement on the pre-allocation scheme, and the final result of β_{ij} was obtained:

$$\mathrm{B}^q=\begin{bmatrix}\beta_{11}^q & \beta_{12}^q & \cdots & \beta_{1n}^q \\ \vdots & \ddots & & \vdots \\ \beta_{n1}^q & \beta_{n2}^q & \cdots & \beta_{nn}^q\end{bmatrix}$$

Where $\begin{bmatrix}\beta_{11}^q & \beta_{12}^q & \cdots & \beta_{1n}^q\end{bmatrix}=\cdots=\begin{bmatrix}\beta_{n1}^q & \beta_{n2}^q & \cdots & \beta_{nn}^q\end{bmatrix}=\begin{bmatrix}\beta_1 & \beta_2 & \cdots & \beta_n\end{bmatrix}$.

6. Example

Assume that a city is convenient for citizens to take public transportation. The bus companies, the subway company and the card company cooperate to implement the integration of transportation card service. The three companies cooperate to cope with the uncertain market environment, and the benefits of the alliance are uncertain. The three companies' opinions on the value of the sub-alliance's income are consistent, but the opinions on the value of the alliance's income are inconsistent. The bus company, the subway company and the card company are represented by 1, 2 and 3 respectively. And the characteristic functions are as follows:

$$v(\phi)=0,\ v(\{1\})=8,\ v(\{2\})=15,\ v(\{3\})=10,\ v_1(N)=30,\ v_2(N)=60,\ v_3(N)=45$$

(1)Let the income of the alliance be $v(N)$, and the income distribution of the company be $y_i=\alpha_i+\beta_i B$, where α_i is the deterministic income, β_i is the distribution coefficient of the uncertain income ($\sum_{i=1}^{3}\beta_i = 1$ and $0 \le \beta_i \le 1$). Since the alliance revenue is completely uncertain, $\sum_{i=1}^{3}\alpha_i = 0$.

Because it is necessary to satisfy individual rationality. The income distribution of company i is not less than $v(i)$, then $y_1 \ge 8$, $y_2 \ge 15$, $y_3 \ge 10$. That is, the constraint is:

$$s.t.\begin{cases} \alpha_1+\alpha_2+\alpha_3=0 \\ \beta_1+\beta_2+\beta_3=1 \\ \alpha_1+30\beta_1 \ge 8 \\ \alpha_2+60\beta_2 \ge 15 \\ \alpha_3+45\beta_3 \ge 10 \\ 0 \le \beta_1,\beta_2,\beta_3 \le 1 \end{cases}$$

According to the above constraints, the feasible domain of (α_i, β_i) is calculated as shown in Figs. 1.

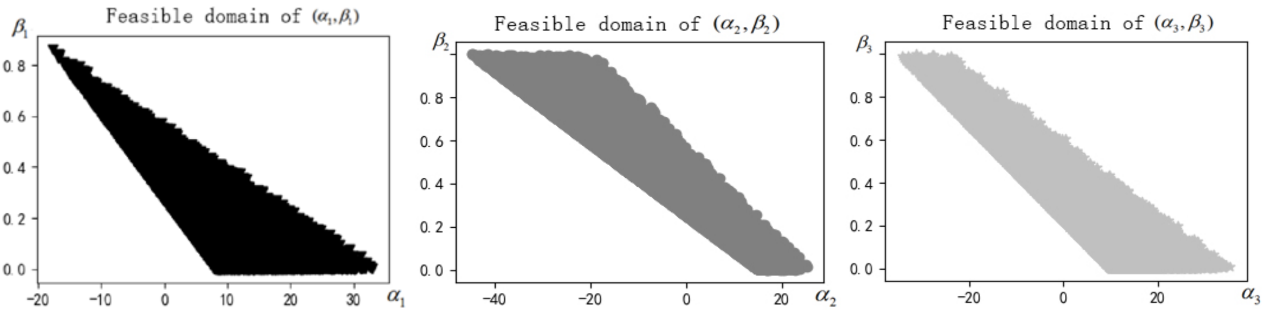


Fig.1 Feasible domain space of (α_i, β_i)

It can be seen from the figure that α_i and β_i are mutually constrained, and the overall trend is inversely proportional. Generally, the larger the deterministic income α_i is, the smaller the uncertainty distribution coefficient β_i is.

(2) Stage 1. Negotiate the deterministic income α_i first.

According to their own preferences and judgments, the three companies put forward initial distribution schemes that reflect their own preferences within the feasible region. The initial distribution matrix of deterministic income is:

$$A^1 = \begin{bmatrix} 15.8235 & -5.0614 & -10.7621 \\ 5.6572 & -20.6878 & 15.0306 \\ 6.0569 & -23.0405 & 16.9836 \end{bmatrix}$$

Initialize various parameters of particle swarm optimization. Set the initial and final values of learning factor $c_{1s}=0.8$, $c_{1e}=0.2$, $c_{2s}=0.4$, $c_{2e}=0.9$, inertia weight $w_s=0.7$, $w_e=0.2$, and maximum iteration number $m=1000$. Particle swarm optimization is used for iterative calculation. The change of pre-distribution value of deterministic income of each company is shown in figure 2:

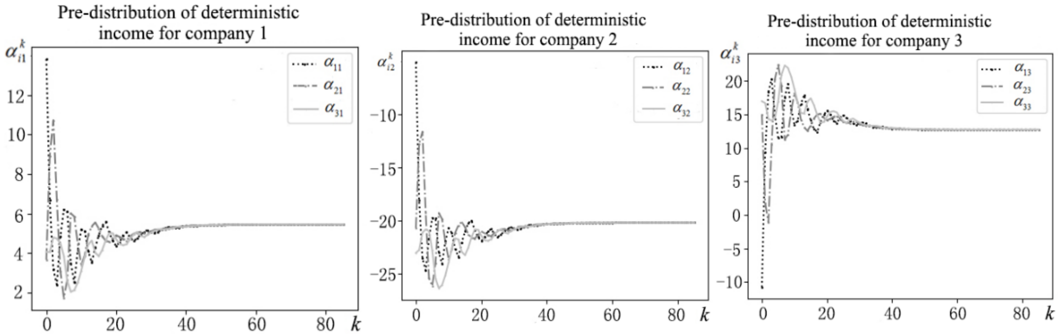


Fig.2 The change chart of definite income pre-distribution value of the three companies

The change chart of deterministic income pre-distribution obviously reflects the iterative negotiation process of each round of deterministic income. The pre-distribution scheme is constantly adjusted, gradually converged and finally reached an agreement. After 86 rounds of iteration, the pre-allocation scheme of each company is agreed, and the result is:

$$A^{86} = \begin{bmatrix} 7.4433 & -20.1778 & 12.7345 \\ 7.4433 & -20.1778 & 12.7345 \\ 7.4433 & -20.1778 & 12.7345 \end{bmatrix}$$

i.e. $\alpha_1 = 7.4433, \alpha_2 = -20.1778, \alpha_3 = 12.7345$

Stage 2. Negotiate the uncertainty distribution coefficient β_i.

Substituting $a_1 = 7.4433, a_2 = -20.1778, a_3 = 12.7345$ into the constraint, and calculating the feasible domain of β_i is shown in Figure 3:

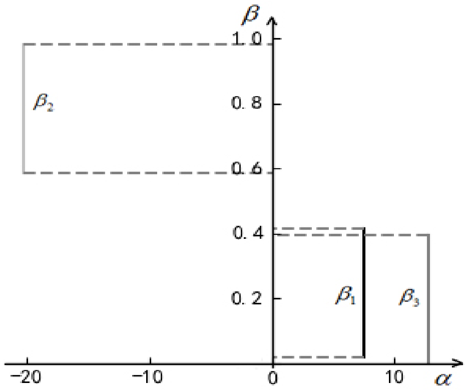


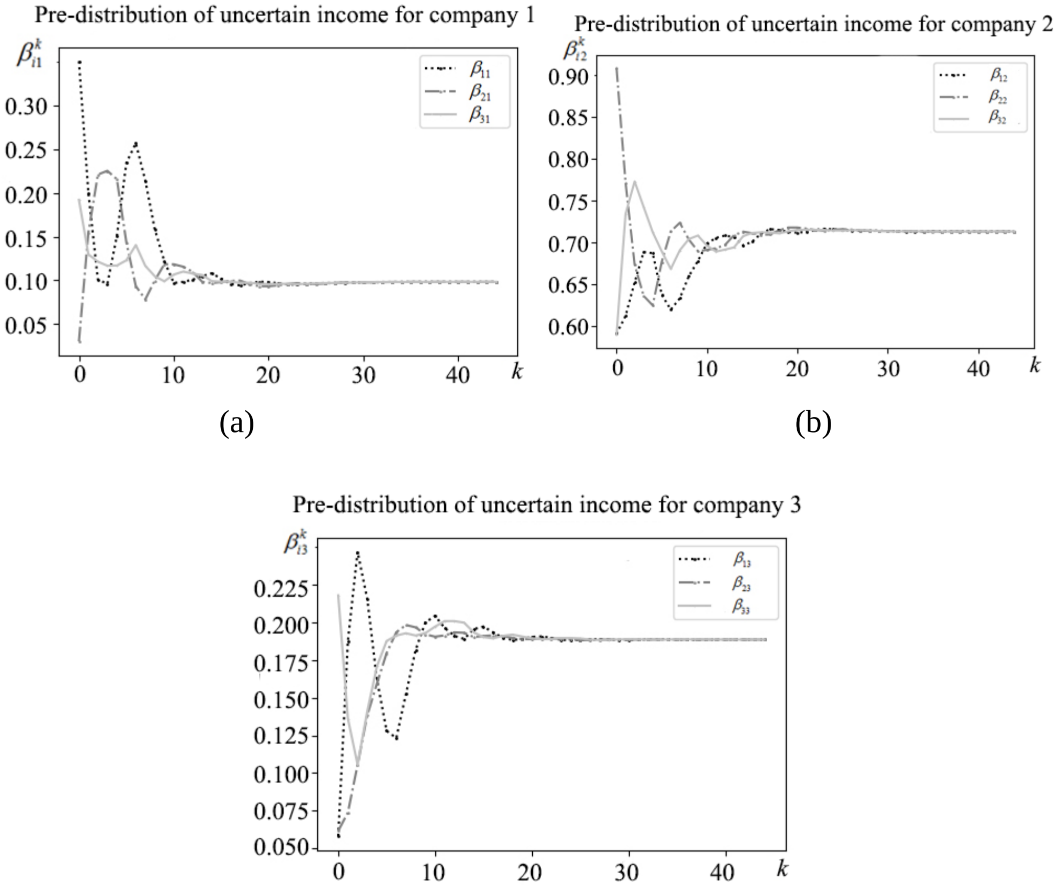
Fig.3 Feasible domain of β_i

i.e. $\beta_1 \in [0.0186, 0.4137]$, $\beta_2 \in [0.5863, 0.9814]$, $\beta_3 \in [0, 0.3951]$

The three companies propose their initial schemes in the feasible domain. The initial allocation matrix of the distribution coefficients is:

$$B^{1} = \begin{bmatrix} 0.3508 & 0.5910 & 0.0582 \\ 0.0302 & 0.9082 & 0.0616 \\ 0.1922 & 0.5900 & 0.2178 \end{bmatrix}$$

Initialize various parameters of particle swarm optimization. Set the initial and final values of learning factor $c_{1s} = 0.8$, $c_{1e} = 0.3$, $c_{2s} = 0.3$, $c_{2e} = 0.8$, inertia weight $w_s = 0.6$, $w_e = 0.2$, and maximum iteration number $m = 1000$. The change of pre-distribution value of uncertainty distribution coefficient of each company is shown in figure 4:



(a) (b)

(c)

Fig.4 Change chart of pre-distribution of uncertain income distribution coefficient

The pre-distribution change graph of the uncertainty income distribution coefficient clearly reflects the process of each company's iterative negotiation of uncertainty income in each round. The pre-allocation scheme is continuously adjusted, gradually converges, and finally reach an agreement. After 45 iterations, the pre-allocation schemes of the companies reached an agreement, and the final result of B is:

$$\mathrm{B}^{45}=\begin{bmatrix}0.0983 & 0.7133 & 0.1884\\0.0983 & 0.7133 & 0.1884\\0.0983 & 0.7133 & 0.1884\end{bmatrix}$$

i.e. $b_1=0.0983, b_2=0.7133, b_3=0.1884$

(3) Finally, the final income distribution is obtained:

$$\begin{cases}y_1=7.4433+0.0983B\\y_2=-20.1778+0.7133B\\y_3=12.7345+0.1884B\end{cases}$$

①When the future actual income of the alliance is $v(N)=35$, the final allocation plan is: $Y=(y_1,y_2,y_3)=(10.8838,4.7877,19.3285)$, the proportion of the income distributed by each enterprise and the actual income: $\Psi=(\psi_1,\psi_2,\psi_3)=(0.3110,0.1368,0.5522)$;

②When the future actual income of the alliance is $v(N)=50$, the final allocation plan is: $Y=(y_1,y_2,y_3)=(12.3583,15.4872,22.1545)$, the proportion of the income distributed by each enterprise and the actual income: $\Psi=(\psi_1,\psi_2,\psi_3)=(0.2472,0.3097,0.4431)$;

③When the future actual income of the alliance is $v(N)=80$, the final allocation plan is: $Y=(y_1, y_2, y_3)=(15.3073,36.8862,27.8065)$, the proportion of the income distributed by each enterprise and the actual income: $\Psi=(\psi_1, \psi_2, \psi_3)=(0.1913,0.4611,0.3476)$;

7. Conclusion

In reality, natural persons build alliances to cope with the future, whereas, natural persons have differences in individual experience, individual information, individual rationality, situation judgment, interest appeal and many other aspects, so it is difficult to form a consensus on the future benefits of the alliance. In this regard, this paper proposes a solution to the cooperative game when the opinions of the alliance's income are inconsistent. Firstly, we consider the income of the player into two parts: deterministic income and uncertain income. Then based on the particle swarm algorithm, consider the group rationality and individual rationality, and a cooperative game particle swarm optimization algorithm model is established. The deterministic income and uncertainty income were solved separately, and get the final allocation scheme. Through the analysis of the example, we can know that when the alliance income is uncertain, we can establish an allocation scheme that can reflect the individual differences of the players. Based on the actual income of the scheme and the future of the alliance, the specific income of the players will be calculated. Compared with the classic solution concept of cooperative game, this is in line with the actual situation and more realistic. The research in this paper provides new ideas and methods for the cooperative game solving of the alliance income uncertainty.

References

[1] Von Neumann, J., & Morgenstern, O. (1944). Theory of Games and Economic Behavior. Princeton, 1944. On Decision-making under uncertainty, 285.

[2] Shapley, L. S. (1953). A value for n-person games. Contributions to the Theory of Games, 2(28), 307-317.

[3] Gillies, D. (1953). Some theorems on n-person games. Ph. D. Dissertation, Princeton University, Department of Mathematics.

[4] Aumann, R. J., & Maschler, M. (1961). The bargaining set for cooperative games. Princeton University.

[5] Davis, M., & Maschler, M. (1965). The kernel of a cooperative game. Naval Research Logistics Quarterly, 12(3), 223-259.

[6] Schmeidler, D. (1969). The nucleolus of a characteristic function game. SIAM Journal on applied mathematics, 17(6), 1163-1170.

[7] Tijs, S. (1981). Bounds for the core and the τ-value. O. Moeschlin and P. Pallaschke, eds., Game Theory and Mathematical Economics.

[8] Tijs, S. H. (1987). An axiomatization of the τ-value. Mathematical Social Sciences, 13(2), 177-181.

[9] Harsanyi, J. C. (1995). Games with incomplete information. The American Economic Review, 85(3), 291-303.

[10] Yang, X., & Gao, J. (2014). Uncertain core for coalitional game with uncertain payoffs. Journal of Uncertain Systems, 8(2), 13-21.

[11] Gao, J., Yang, X., & Liu, D. (2017). Uncertain Shapley value of coalitional game with application to supply chain alliance. Applied Soft Computing, 56, 551-556.

[12] Algaba, E., Fragnelli, V., Llorca, N., & Sánchez-Soriano, J. (2019). Horizontal cooperation in a multimodal public transport system: The profit allocation problem. European Journal of Operational Research, 275(2), 659-665.

[13] Eberhart, R., & Kennedy, J. (1995, November). Particle swarm optimization. In Proceedings of the IEEE international conference on neural networks (Vol. 4, pp. 1942-1948).

[14] Clerc, M. (2010). Particle swarm optimization (Vol. 93). John Wiley & Sons.

[15] Shi, Y., & Eberhart, R. (1998, May). A modified particle swarm optimizer. In 1998 IEEE international conference on evolutionary computation proceedings. IEEE world congress on computational intelligence (Cat. No. 98TH8360) (pp. 69-73). IEEE.

[16] Ratnaweera, A., Halgamuge, S. K., & Watson, H. C. (2004). Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients. IEEE Transactions on evolutionary computation, 8(3), 240-255.